

H. T. DIEP

Professor
University of Cergy-Pontoise, France

STATISTICAL PHYSICS

Fundamentals and Application to Condensed Matter

Lectures, Problems and Solutions



ARTISTIC VIEW OF A VORTEX

BOOK, 647 PAGES, 15 CHAPTERS, TO APPEAR IN APRIL 2015, BY WORLD SCIENTIFIC

To my wife and our children Samuel, Tuan, Kim and Sarah.

To my mother.

Foreword

Statistical mechanics provides general methods to study properties of systems composed of a large number of particles. It establishes general formulas connecting physical parameters to various physical quantities. When parameters of a system such as interaction between particles and temperature are known, one can deduce its macroscopic properties. In general, microscopic mechanisms leading to interactions are provided by quantum mechanics. The combination of statistical mechanics and quantum mechanics has provided an understanding of properties of matter leading to spectacular technological innovations and discoveries which have radically changed our daily life since the sixties.

This book is based on the author's lectures for students of the third year of the Bachelor's degree in Physics. The second part of the book treats selected advanced subjects taught at the Master's level. In each chapter, fundamental notions and techniques are presented and followed by applications chosen among frequently encountered phenomena. Demonstrations leading to main results are given in details.

In the first part, after an introduction of basic definitions and mathematical tools (chapter 1), the book covers the foundation of statistical physics at equilibrium: starting from the fundamental postulate, the book deals with systems under various situations going from isolated systems (chapter 2) and systems maintained at a constant temperature (chapter 3) to open systems (chapter 4). The main properties of free fermions (chapter 5) and free bosons (chapter 6) are studied to a great extent. The first part ends with chapter 7 where the method of second quantization is shown. This method, though conceptually more abstract than the Schrödinger equation, is technically less cumbersome to handle, and is very useful in the study of weakly interacting many-particle systems. A large number of applications

of this method is found in the remaining chapters.

In the second part, advanced techniques and applications in condensed matter are presented. Selected topics in condensed matter include vibrations of atoms in crystals, conducting electrons in metals and superconductors, band structures in semiconductors, and magnetic properties of materials. Statistical physics contributes with quantum mechanics to the success of these fields in the last fifty years. In chapter 8 the crystalline symmetry is presented with all necessary notions for studying properties of electrons and atoms in crystals. In chapter 9 systems of interacting atoms in crystals are considered. Quantized atom vibrations are called phonons which dominate thermodynamic properties of solids. Systems of interacting conducting electrons are studied in chapter 10 along with general properties of Fermi liquids. The origin of energy bands of electrons in semiconductors is shown in chapter 11. Conducting electrons are at the heart of charge and spin transport phenomena with an enormous number of applications. The spin carried by an electron plays thus a very important role in condensed matter physics. Magnetic properties due to spins cannot be separated from other properties of matter. Note that systems of interacting spins constitute one of the most important subjects in statistical physics. They are studied in chapter 12 where collective excitations, called spin waves or magnons, are shown in details. The abundance of magnetic materials, natural or artificial compounds, provides an inexhaustible source of applications. Chapter 13 deals with the phase transition in spin systems where basic notions such as symmetry breaking and universality class are introduced. The mean-field approximation and the Landau-Ginzburg theory for second-order phase transitions are presented. The concept of the renormalization group is described. In chapter 14, the Ginzburg-Landau theory for the superconductivity is developed to explain properties of type I and type II superconductors. The microscopic Bardeen-Cooper-Schrieffer theory for conventional superconductors is presented in details in this chapter. The second part of the book ends with chapter 15 where basic notions on systems out of equilibrium and the Boltzmann's equation are introduced. As applications of the Boltzmann's equation, properties of electron transport in metals and semiconductors are studied to a great extent.

In the third part of the book, solutions of problems are given. These problems are conceived for self-training and to help the reader discover new related phenomena which are complementary to the lectures.

H. T. Diep, Professor of Physics, University of Cergy-Pontoise, France.

Acknowledgments

I am grateful to my many colleagues at the University of Cergy-Pontoise for sharing uncountable stimulating moments in my professional life and for their precious friendship during the last 25 years. My sincere gratitude goes to all the people who have contributed in one way or another to forging the site into the education and research institution that it is today. Thanks to them, it is the place where I go to work every day with enthusiasm and eagerness.

I am in particular indebted to numerous administrative staffs who have been working with me over the years, for their generosity and for carrying out with me our collective duties in joy and mutual trust. I am proud of what we have achieved together. It was always a team effort which is key in overcoming obstacles and fighting adversity.

I would like to express here my deep affection for my former and current doctorate students with whom I shared innumerable wonderful moments not only in our research activities but also in discussions on many subjects of life.

H. T. Diep

Contents

<i>Foreword</i>	vii
<i>Acknowledgments</i>	ix
<i>List of Problems</i>	xxi
Fundamentals of Statistical Physics	3
1. Basic Concepts and Tools in Statistical Physics	5
1.1 Introduction	5
1.2 Combinatory analysis	6
1.2.1 Number of permutations	6
1.2.2 Number of arrangements	6
1.2.3 Number of combinations	7
1.3 Probability	7
1.3.1 Definition	7
1.3.2 Fundamental properties	8
1.3.3 Mean values	9
1.4 Statistical distributions	11
1.4.1 Binomial distribution	11
1.4.2 Gaussian distribution	12
1.4.3 Poisson law	14
1.5 Microstates - Macrostates	15
1.5.1 Microstates - Enumeration	15
1.5.2 Macroscopic states	17
1.5.3 Statistical averages - Ergodic hypothesis	17
1.6 Statistical entropy	18

1.7	Conclusion	19
1.8	Problems	19
2.	Isolated Systems: Micro-Canonical Description	23
2.1	Introduction	23
2.2	Fundamental postulate	23
2.3	Properties of an isolated system	25
2.3.1	Spontaneous evolution of an isolated system toward equilibrium	25
2.3.2	Exchanges of heat and volume	26
2.3.3	Exchange of particles	27
2.3.4	Statistical distribution of an internal variable	27
2.4	Phase space - Density of states	29
2.4.1	Density of states	29
2.4.2	Density of states of free quantum particles	30
2.4.3	Density of states of free classical particles	32
2.5	Applications of the micro-canonical method	33
2.5.1	Example 1: two-level systems	34
2.5.2	Example 2: Classical ideal gas	36
2.6	Conclusion	37
2.7	Problems	37
3.	Systems at a Constant Temperature: Canonical Description	45
3.1	Canonical probability	45
3.2	Partition function	47
3.3	Properties of a system at a constant temperature	48
3.4	Statistical distribution of an internal variable	50
3.5	Spontaneous evolution of a canonical system	51
3.5.1	Criterion for equilibrium	51
3.5.2	Direction of spontaneous evolution	53
3.6	Applications of the canonical method	54
3.6.1	Systems of identical independent particles	54
3.6.2	Classical ideal gas	56
3.6.3	Two-level systems	57
3.6.4	Theorem of equipartition of energy	58
3.7	Conclusion	60
3.8	Problems	61
4.	Open Systems at Constant Temperature: Grand-	

Canonical Description	67
4.1 Introduction	67
4.2 Grand-canonical probability	67
4.3 Grand partition function \mathcal{Z} - Grand potential J	69
4.4 General properties of grand-canonical systems	70
4.5 Spontaneous evolution of a grand-canonical system	72
4.6 Systems of identical, independent particles	74
4.6.1 Factorization of \mathcal{Z}	75
4.6.2 Bose-Einstein distribution	76
4.6.3 Fermi-Dirac distribution	76
4.6.4 Maxwell-Boltzmann distribution	77
4.7 Applications of the grand-canonical method	77
4.7.1 Classical ideal gas	77
4.7.2 Two-level systems	78
4.8 Conclusion	80
4.9 Problems	80
5. Free Fermi Gas	85
5.1 Introduction	85
5.2 Fermi-Dirac distribution	85
5.3 General properties of a free Fermi gas	86
5.3.1 General formulas	86
5.3.2 Formulas for large systems	88
5.4 Properties of a free Fermi gas at $T = 0$	90
5.4.1 Fermi energy	90
5.4.2 Total average kinetic energy	91
5.5 Properties of a free Fermi gas at low temperatures	91
5.5.1 Sommerfeld's expansion	91
5.5.2 Chemical potential, average energy and calorific capacity	92
5.6 Free Fermi gas at the high-temperature limit	92
5.7 Applications	93
5.7.1 Paramagnetism of conducting electrons in metals	93
5.7.2 Thermo-ionic emission	95
5.8 Conclusion	96
5.9 Problems	96
6. Free Boson Gas	99

6.1	Introduction	99
6.2	Bose-Einstein distribution	99
6.3	General properties of a free boson gas	100
	6.3.1 General formulas	100
	6.3.2 Formulas for large systems	102
6.4	High-temperature limit	103
6.5	Bose-Einstein condensation	104
6.6	Properties at temperatures higher than T_B	105
6.7	Applications	108
	6.7.1 Photons: black-body radiation	108
	6.7.2 Helium-4	112
6.8	Conclusion	113
6.9	Problems	113
7.	Systems of Interacting Particles: Method of Second Quantization	115
7.1	Introduction	115
7.2	First quantization: symmetric and antisymmetric wave functions	116
7.3	Representation of microstates by occupation numbers	120
7.4	Second quantization: the case of bosons	121
	7.4.1 Hamiltonian in second quantization	121
	7.4.2 Properties of boson operators	125
7.5	Second quantization: the case of fermions	126
7.6	Field operators	129
7.7	Hartree-Fock approximation	131
7.8	Conclusion	134
7.9	Problems	134
	Application to Condensed Matter	137
8.	Symmetry in Crystalline Solids	139
8.1	Crystalline symmetry	139
8.2	Reciprocal lattices	141
8.3	Wave-vector space - Brillouin zones	144
8.4	Sum rules	146
8.5	Fourier analysis	148
8.6	Representation in \vec{k} -space	150
8.7	Conclusion	150

8.8	Problems	150
9.	Interacting Atoms in Crystals: Phonons	153
9.1	Introduction	153
9.2	Vibrations in one dimension	155
9.2.1	Equation of motion	155
9.2.2	Dispersion relation	156
9.3	Vibrations in two and three dimensions	157
9.4	Quantization of vibration: phonons	161
9.4.1	Normal coordinates, vibration energy	161
9.4.2	Quantization of vibration	162
9.5	Thermal properties of phonons	164
9.5.1	Density of modes	165
9.5.2	Einstein model and Debye model	166
9.6	Phonons in a condensed gas of Helium-4	170
9.7	Conclusion	174
9.8	Problems	175
10.	Systems of Interacting Electrons - Fermi Liquids	179
10.1	Introduction	179
10.2	Gas of interacting electrons	179
10.2.1	Kinetic and exchange energies	183
10.2.2	Effective mass	186
10.3	Gas of interacting electrons by second quantization	186
10.3.1	Kinetic energy	189
10.3.2	Energy at first-order perturbation	190
10.3.3	Energy at second-order perturbation	191
10.4	Fermi Liquids	195
10.5	Kondo effect	197
10.6	Conclusion	198
10.7	Problems	198
11.	Electrons in Crystalline Solids: Energy Bands	203
11.1	Wave function of an electron in a periodic potential: Bloch function	204
11.2	Theory of almost-free electrons	206
11.2.1	One-dimensional case	206
11.2.2	Calculation of the energy correction	209

11.2.3	Interpretation of the forbidden band gap	211
11.2.4	Three-dimensional case	212
11.3	Electrons in a periodic potential: the central equation . .	214
11.3.1	Band filling: classification of materials	216
11.3.2	Semiconductors	217
11.4	Tight-Binding Approximation	221
11.4.1	One-dimensional case	221
11.4.2	Three-dimensional case	226
11.4.3	Velocity, acceleration, effective mass	227
11.5	Conclusion	228
11.6	Problems	230
12.	Systems of Interacting Spins: Magnons	237
12.1	Spin models	237
12.1.1	Heisenberg model	237
12.1.2	Ising, XY and Potts models	239
12.2	Spin waves in ferromagnets	241
12.2.1	Classical treatment	241
12.2.2	Quantum theory	246
12.2.3	Properties at low temperatures	249
12.3	Other magnets	252
12.3.1	Antiferromagnets	252
12.3.2	Ferrimagnets	252
12.3.3	Helimagnets	252
12.3.4	Frustrated magnets	254
12.4	Conclusion	256
12.5	Problems	256
13.	Systems of Interacting Spins: Phase Transitions	261
13.1	Introduction	261
13.2	Generalities	262
13.2.1	Order parameter	262
13.2.2	Order of the phase transition	264
13.2.3	Correlation function - Correlation length	264
13.2.4	Critical exponents	265
13.2.5	Universality class	266
13.3	Ferromagnetism in mean-field theory	268
13.3.1	Mean-field equation	268

13.3.2	Critical temperature	271
13.3.3	Specific heat	273
13.3.4	Susceptibility	274
13.3.5	Validity of mean-field theory	276
13.3.6	Improved mean-field theory: Bethe's approximation	276
13.4	Landau-Ginzburg theory	278
13.4.1	Mean-field critical exponents	279
13.4.2	Correlation function	280
13.4.3	Corrections to mean-field theory	282
13.5	Renormalization group	284
13.5.1	Transformation of renormalization group - Fixed point	284
13.5.2	Renormalization group applied to an Ising chain .	287
13.5.3	Migdal-Kadanoff decimation method and Migdal- Kadanoff bond-moving approximation	289
13.6	Transfer matrix method applied to an Ising chain	293
13.7	Phase transition in some particular systems	295
13.7.1	Exactly solved spin systems	295
13.7.2	Kosterlitz-Thouless transition	296
13.7.3	Frustrated spin systems	296
13.8	Conclusion	297
13.9	Problems	297
14.	Superconductivity	303
14.1	Introduction	303
14.2	Properties of conventional superconductors	304
14.3	Ginzburg-Landau theory of superconductivity	305
14.4	Superconductors of type II	312
14.5	Bardeen-Cooper-Schrieffer theory	316
14.5.1	Electron-phonon interaction	316
14.5.2	Cooper electron pairs- BCS Hamiltonian	319
14.5.3	Ground-state wave function	323
14.6	High-temperature superconductivity	325
14.7	Conclusion	326
14.8	Problems	327
15.	Transport in Metals and Semiconductors	329
15.1	Introduction	329

15.2	Boltzmann's equation	329
15.2.1	Classical formulation	330
15.2.2	Quantum formulation	331
15.3	Linearized Boltzmann's equation	333
15.3.1	Explicit linearized terms	333
15.3.2	Relaxation-time approximation	334
15.4	Applications in general transport problems	335
15.4.1	Heat current	335
15.4.2	Thermo-electric current	335
15.4.3	Electric conductivity	338
15.4.4	Thermal conductivity	338
15.4.5	Seebeck effect	339
15.4.6	Peltier effect	339
15.5	Resistivity	339
15.6	Spin-independent transport in metals - Ohm's law	340
15.7	Transport in strong electric fields - Hot electrons	342
15.8	Transport in semiconductors	347
15.8.1	Motion of electrons in applied fields - Hall effect	347
15.8.2	Calculation of the diffusion coefficient by the Boltzmann's equation	352
15.8.3	Transport in semiconductors: Gunn's effect	355
15.8.4	Conductivity in extrinsic semiconductors - Doping effects	359
15.8.5	Doped semiconductors: generation, recombination, equation of continuity	362
15.8.6	$p - n$ junctions - Diodes	365
15.9	Spin transport in magnetic materials	369
15.10	Conclusion	370
15.11	Problems	370

Solutions of Problems 377

<i>Solutions of Problems of Part 1</i>	379
16.1 Solutions of problems of chapter 1	379
16.2 Solutions of problems of chapter 2	387
16.3 Solutions of problems of chapter 3	408
16.4 Solutions of problems of chapter 4	424
16.5 Solutions of problems of chapter 5	439

16.6	Solutions of problems of chapter 6	451
16.7	Solutions of problems of chapter 7	458
<i>Solutions of Problems of Part 2</i>		467
17.1	Solutions of problems of chapter 8	467
17.2	Solutions of problems of chapter 9	471
17.3	Solutions of problems of chapter 10	483
17.4	Solutions of problems of chapter 11	497
17.5	Solutions of problems of chapter 12	509
17.6	Solutions of problems of chapter 13	525
17.7	Solutions of problems of chapter 14	537
17.8	Solutions of problems of chapter 15	550
Appendices		573
Appendix A	Mathematical Complements and Table of Constants	575
A.1	Volume of a sphere in n dimensions	575
A.2	Stirling formula	576
A.3	Gaussian integrals	576
A.4	Γ function	577
A.5	ζ series or Riemann's series	577
A.6	Other formulas	577
A.7	Universal constants	578
Appendix B	Sommerfeld's Expansion at Low Temperatures	579
Appendix C	Origin of the Heisenberg Model	581
Appendix D	Hubbard Model: Superexchange	585
Appendix E	Kosterlitz-Thouless Phase Transition	593
Appendix F	Low- and High-Temperature Expansions of the Ising Model	601
F.1	The case of the square lattice	601
F.2	The case of the triangular and honeycomb lattices	606
<i>Bibliography</i>		611
<i>Index</i>		619

List of Problems

Problems of chapter 1: Basic Concepts and Tools in Statistical Physics

- Problem 1. Central limit theorem
- Problem 2. Poisson law (1.32)
- Problem 3. Demonstration of the formulas (1.34) and (1.35)
- Problem 4. Application of the binomial law
- Problem 5. Random walk in one dimension
- Problem 6. Random walk in three dimensions
- Problem 7. Exchange of energy
- Problem 8. Statistical entropy

Problems of chapter 2: Isolated Systems - Micro-Canonical Description

- Problem 1. Joule expansion
- Problem 2. Exchange of heat
- Problem 3. Distribution of energy on particles
- Problem 4. System of magnetic moments in an applied magnetic field
- Problem 5. Density of states in one and two dimensions
- Problem 6. Classical ideal gas in one and two dimensions
- Problem 7. Classical ideal gas
- Problem 8. Classical harmonic oscillator
- Problem 9. System of classical harmonic oscillators
- Problem 10. System of quantum harmonic oscillators
- Problem 11. Subsystems of quantum harmonic oscillators
- Problem 12. Frenkel's defects by micro-canonical method
- Problem 13. Schottky's defects by micro-canonical method
- Problem 14. Exchange of heat in three dimensions
- Problem 15. Binary alloy

Problems of chapter 3: Systems at a Constant Temperature - Canonical Description

- Problem 1. Calorific capacity C_V
- Problem 2. Maxwell-Boltzmann approximation
- Problem 3. Bi-dimensional classical ideal gas
- Problem 4. Classical harmonic oscillators
- Problem 5. Quantum harmonic oscillators
- Problem 6. Three-level system
- Problem 7. Frenkel's defects by canonical method
- Problem 8. Schottky's defects by canonical method
- Problem 9. Velocity distribution in a classical ideal gas
- Problem 10. System of spins in an applied magnetic field
- Problem 11. Equilibrium of a vapor-solid system
- Problem 12. Harmonic oscillators

Problems of chapter 4: Open Systems at Constant Temperature: Grand-Canonical Description

- Problem 1. Fluctuations of N
- Problem 2. Classical ideal gas in the gravitational field
- Problem 3. Two-level system
- Problem 4. Degeneracy in the case of fermions
- Problem 5. Particle trap
- Problem 6. Poisson law by the grand-canonical description
- Problem 7. System of interacting electrons
- Problem 8. Lattice model for an ideal gas
- Problem 9. Adsorption
- Problem 10. Adsorption of an ideal gas on the surface of a solid

Problems of chapter 5: Free Fermi Gas

- Problem 1. Free Fermi gas at thermodynamic limit
- Problem 2. Fermi gas at low temperatures
- Problem 3. Free Fermi gas in one and two dimensions
- Problem 4. Free Fermi gas in two dimensions
- Problem 5. Pressure of a free Fermi gas
- Problem 6. Free Fermi gas with internal degrees of freedom
- Problem 7. Ultra relativistic ideal gas
- Problem 8. Electrons in Sodium
- Problem 9. Pauli paramagnetism

Problem 10. Electrons in Copper

Problems of chapter 6: Free Boson Gas

Problem 1. Gas of photons

Problem 2. Bose-Einstein condensation

Problem 3. Pressure in a gas of bosons

Problem 4. Two-dimensional gas of bosons

Problem 5. Gas of bosons with internal degrees of freedom

Problem 6. Einstein's model for the vibration of atoms on a lattice

Problem 7. Wien displacement law

Problem 8. Equation of state of a gas of photons

Problems of chapter 7: Systems of Interacting Particles - Method of Second Quantization

Problem 1. Exercise of boson and fermion operators

Problem 2. Exercise on commutation relations of field operators

Problem 3. Exercise of field operators

Problem 4. Exercise of field operators

Problem 5. Boson Hamiltonian

Problem 6. Gas of interacting bosons

Problem 7. Diagonalization of Hamiltonian in second quantization

Problems of chapter 8: Symmetry in Crystalline Solids

Problem 1. Reciprocal lattice of a triangular lattice

Problem 2. Honeycomb lattice

Problem 3. Face-centered cubic lattice - Body-centered cubic lattice

Problem 4. Chain of two types of atom

Problem 5. Periodic potential

Problem 6. Fourier transform of the Coulomb potential

Problem 7. Fourier analysis

Problem 8. Structure factors

Problems of chapter 9: Interacting Atoms in Crystals - Phonons

Problem 1. Demonstration of (9.36)

Problem 2. Chain of two types of atoms

Problem 3. Interaction between next nearest neighbors in a chain

Problem 4. Chain of two types of distance

Problem 5. Phonons in a rectangular lattice

- Problem 6. Phonons in a simple cubic lattice
- Problem 7. Density of modes in a square lattice
- Problem 8. Energy and specific heat at finite temperatures
- Problem 9. Phonons in a chain with long-range interaction
- Problem 10. Phonons and melting

Problems of chapter 10: Systems of Interacting Electrons - Fermi Liquids

- Problem 1. System of two electrons - Fermi hole
- Problem 2. Screened Coulomb potential, Thomas-Fermi approximation
- Problem 3. Hartree-Fock approximation
- Problem 4. Paramagnetic-ferromagnetic transition in an electron gas
- Problem 5. Gas of interacting fermions in second quantization
- Problem 6. Fermion gas as a function of density

Problems of chapter 11: Electrons in Crystalline Solids - Energy Bands

- Problem 1. Perturbation near the boundary of a Brillouin zone
- Problem 2. Electrons in a square lattice
- Problem 3. Electrons in a rectangular lattice
- Problem 4. Energy band in the Kronig-Penney potential model
- Problem 5. Tight-binding approximation in one dimension
- Problem 6. Tight-binding approximation in a square lattice
- Problem 7. Tight-binding approximation in a face-centered cubic lattice

Problems of chapter 12: Systems of Interacting Spins - Magnons

- Problem 1. Ground-state spin configuration of a chain of Ising spins
- Problem 2. Chain of Heisenberg spins
- Problem 3. Commutation relations
- Problem 4. Heisenberg model in two dimensions
- Problem 5. Magnon-phonon interaction
- Problem 6. Magnons in antiferromagnets
- Problem 7. Properties at low temperatures of antiferromagnets
- Problem 8. Magnons in helimagnets
- Problem 9. Triangular antiferromagnet
- Problem 10. Villain's model

Problems of chapter 13: Systems of Interacting Spins - Phase Transitions

- Problem 1. Ising spin model in the mean-field approximation

- Problem 2. Next-nearest-neighbor interaction in a centered cubic lattice
- Problem 3. Next-nearest-neighbor interaction in a square lattice
- Problem 4. System of two spins
- Problem 5. Improved mean-field approximation: Bethe's approximation
- Problem 6. Chain of Ising spins by micro-canonical method
- Problem 7. Chain of Ising spins by canonical method
- Problem 8. Mean-field approximation for antiferromagnets
- Problem 9. Ferrimagnets by mean-field theory
- Problem 10. Chain of Ising spins by exact method

Problems of chapter 14: Superconductivity

- Problem 1. Demonstration of Ginzburg-Landau Eqs. (14.13) and (14.14)
- Problem 2. Current density \vec{J} : gauge-invariance
- Problem 3. Theory of Gorter-Casimir
- Problem 4. Energy of a vortex
- Problem 5. Electron gas in a strong magnetic field: Landau's levels

Problems of chapter 15: Transport in Metals and Semiconductors

- Problem 1. Effect of magnetic field: Demonstration of Eq. (15.24)
- Problem 2. Electrons in a strong electric field: an approximation
- Problem 3. Semiconductors: effect of temperature on conductivity
- Problem 4. Semiconductor: effect of magnetic field on the gap
- Problem 5. Effect of doping in semiconductors
- Problem 6. Shallow impurity states in semiconductors
- Problem 7. Recombinations in semiconductors
- Problem 8. Dielectric relaxation
- Problem 9. Polarized $p - n$ junction: direct current
- Problem 10. Transport in a superlattice
- Problem 11. Hall effect - Magnetoresistance